# 1 Temperature

## 1.1 micro state

Definition:

$$\frac{1}{k_B T} = \frac{\mathrm{d} \ln \Omega}{\mathrm{d} E} \tag{1.1}$$

where $\Omega$ is the number of microstate, and $k_B$ is Boltzmann constant:

$$k_B = 1.3807 \times 10^{-23}\,\mathrm{J\,K^{-1}} \tag{1.2}$$

$$\frac{1}{T} = \frac{\mathrm{d}S}{\mathrm{d}E} \tag{1.3}$$

hence:

$$\mathrm{d}S = k_B \ln \Omega \tag{1.4}$$

## 1.2 Boltzmann factor

The probability $P(\epsilon)$ of a system at energy $\epsilon$ is:

$$P(\epsilon) \propto e^{-\epsilon/k_B T} \tag{1.5}$$

We define the quantity to simplify:

$$\beta \equiv \frac{1}{k_B T} \tag{1.6}$$

then the Boltzmann factor is $e^{-\beta E}$, and

$$\beta = \frac{\mathrm{d} \ln \Omega}{\mathrm{d} E} \tag{1.7}$$

# 2 Maxwell distribution

## 2.1 velocity distribution

$$g(v_x) = \sqrt{\frac{m}{2\pi k_B T}}\, e^{-m v_x^2/2k_B T} \tag{2.1}$$

## 2.2 speed distribution

For 3 dimensional speed distribution:

$$f(v)dv = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_B T}\right)^{3/2} v^2 \mathrm{d}v\, e^{-mv^2/2k_B T} \tag{2.2}$$

which implies the average and rms speed:

$$\bar{v} = \langle v \rangle = \sqrt{\frac{8k_B T}{\pi m}} \tag{2.3}$$

$$v_{\mathrm{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_B T}{m}} \tag{2.4}$$

# 3 Entropy

$$\mathrm{d}S = \frac{\mathrm{d}Q_{\mathrm{rev}}}{T} \tag{3.1}$$

# 4 Potentials

## 4.1 Internal energy

$$\mathrm{d}U = T\mathrm{d}S - p\mathrm{d}V \tag{4.1}$$

which implies:

$$T = \left(\frac{\partial U}{\partial S}\right)_V, p = -\left(\frac{\partial U}{\partial V}\right)_S \tag{4.2}$$

## 4.2 Enthalpy

Definition:

$$H = U + pV \tag{4.3}$$

then

$$\mathrm{d}H = T\mathrm{d}S + V\mathrm{d}p \tag{4.4}$$

which implies:

$$T = \left(\frac{\partial H}{\partial S}\right)_p, V = \left(\frac{\partial H}{\partial p}\right)_S \tag{4.5}$$

## 4.3 Helmholtz function

Definition:

$$F = U - TS \tag{4.6}$$

then

$$\mathrm{d}F = -S\mathrm{d}T - p\mathrm{d}V \tag{4.7}$$

which implies:

$$S = -\left(\frac{\partial F}{\partial T}\right)_S, p = -\left(\frac{\partial F}{\partial V}\right)_T \tag{4.8}$$

## 4.4 Gibbs function

Definition:

$$G = H - TS \tag{4.9}$$

$$\mathrm{d}H = -S\mathrm{d}T + V\mathrm{d}p \tag{4.10}$$

which implies:

$$S = -\left(\frac{\partial G}{\partial T}\right)_p, V = \left(\frac{\partial G}{\partial p}\right)_T \tag{4.11}$$

# 5 Maxwell's relations

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \tag{5.1}$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p \tag{5.2}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \tag{5.3}$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p \tag{5.4}$$

# 6 Susceptibility

## 6.1 Expansivity

$$\alpha = \beta_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \tag{6.1}$$

## 6.2 Compressibility

$$\kappa = \kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \tag{6.2}$$

# 7 Heat capacity

$$C_V \equiv \left(\frac{\partial Q}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V \tag{7.1}$$

$$C_p \equiv \left(\frac{\partial Q}{\partial T}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p \tag{7.2}$$

$$C_p - C_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{\partial V}{\partial T}\right)_p \tag{7.3}$$

$$= \frac{VT\alpha^2}{\kappa} \tag{7.4}$$

adiabatic index:

$$\gamma \equiv \frac{C_p}{C_V} \tag{7.5}$$

# 8 Partition function

There are some states $\alpha$ in a system. Define the partition function $Z$:

$$Z = \sum_\alpha e^{-\beta E_\alpha} \tag{8.1}$$

which implies:

$$U = -\frac{\mathrm{d}\ln Z}{\mathrm{d}\beta} \tag{8.2}$$

$$S = \frac{U}{T} + k_B \ln Z \tag{8.3}$$

$$F = -k_B T \ln Z \tag{8.4}$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T = k_B T\left(\frac{\partial \ln Z}{\partial V}\right)_T \tag{8.5}$$

## 8.1 Density of states

$$g(k)\mathrm{d}k = \frac{Vk^2\mathrm{d}k}{2\pi^2} \tag{8.6}$$

## 8.2 Quantum concentration

partition function of one particle:

$$Z_1 \equiv \int_0^\infty e^{-\beta E(k)} g(k)\,\mathrm{d}k = V n_Q \tag{8.7}$$

where

$$n_Q = \frac{1}{\hbar^3}\left(\frac{mk_BT}{2\pi}\right)^{3/2} \tag{8.8}$$

and thermal wavelength:

$$\lambda_{\mathrm{th}} = n_Q^{-1/3} = \frac{h}{\sqrt{2\pi m k_B T}} \tag{8.9}$$

## 8.3 $N$ particles

For $N$ indistinguishable particles:

$$Z_N = \frac{(Z_1)^N}{N!} \tag{8.10}$$

which is satisfied when

$$[\text{number density}] \ll n_Q \tag{8.11}$$

# 9 Black body

## 9.1 Plank distribution

$$u_\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{\beta h\nu} - 1} \tag{9.1}$$

## 9.2 Stefan-Boltzmann law

power incident per unit area:

$$J = \sigma T^4 \tag{9.2}$$

where

$$\sigma = \frac{\pi^2 k_B^4}{60c^2\hbar_3} \approx 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}} \tag{9.3}$$

# 10 van der Waals gas

molar volume $V_m = V/n$. Definition:

$$\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT \tag{10.1}$$

critical point satisfies:

$$\left(\frac{\partial p}{\partial V}\right)_T = 0, \left(\frac{\partial^2 p}{\partial T^2}\right)_T = 0 \tag{10.2}$$

hence:

$$V_c = 3b \tag{10.3}$$

$$T_c = \frac{8a}{27Rb} \tag{10.4}$$

$$p_c = \frac{a}{27b^2} \tag{10.5}$$

# 11 Phase transition

## 11.1 Latent heat

When changing from phase 1 to phase 2 at constant temperature $T_c$, it needs "latent heat":

$$L = \Delta Q_{\text{rev}} = T_c(S_2 - S_1) \tag{11.1}$$

## 11.2 Clausius-Clapeyron

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{T(V_2 - V_1)} \tag{11.2}$$

# 12 Bose-Einstein and Fermi-Dirac distribution

## 12.1 bosons and fermions

When exchange particles, if the wave function is symmetric, the particles are called bosons. On the other hand, if it's antisymmetric, they are fermions.

## 12.2 Partion function

$$Z = \sum_n e^{-n\beta E} \tag{12.1}$$

For fermions, because of Pauli exclusion principle, $n = 0, 1$. And for bosons, $n = [0, \infty)$. Therefore:

$$\ln Z = \pm \ln\left(1 \pm e^{-\beta E}\right) \tag{12.2}$$

where + for fermions and − for bosons.

The mean number of particles, same as the distribution function $f(E)$:

$$\langle n \rangle = f(E) = \frac{1}{e^{\beta E} \pm 1} \tag{12.3}$$

where + for Fermi-Dirac and − for Bose-Einstein distribution function.